WL01 WAVES IN 2D

SPH4U

CH 9 (KEY IDEAS)

- analyze and interpret the properties of two-dimensional mechanical waves in a ripple tank and relate them to light
- derive and apply equations involving the speed, wavelength, frequency, and refractive index of waves and apply them to the behaviour light
- analyze two-point-source interference patterns in a ripple tank and in the interference of light (Young's experiment) using diagrams
- derive and apply equations relating the properties of wave interference and wavelength
- outline the historical development of the particle and wave theories of light, including the development of new technologies and discoveries, and summarize the successes and failures of each theory
- apply the wave theory to the property of dispersion and determine the wavelengths of the colours of the visible spectrum

EQUATIONS

- Wave Equations

$$f = \frac{1}{T}$$

- Universal Wave Equation

$$v = f\lambda$$
$$c = f\lambda$$

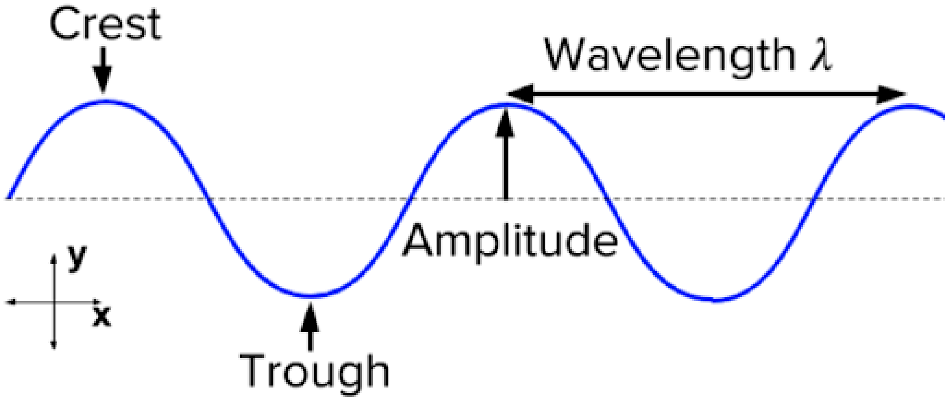
- Snell's Law with Universal Wave Equation

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

RECALL

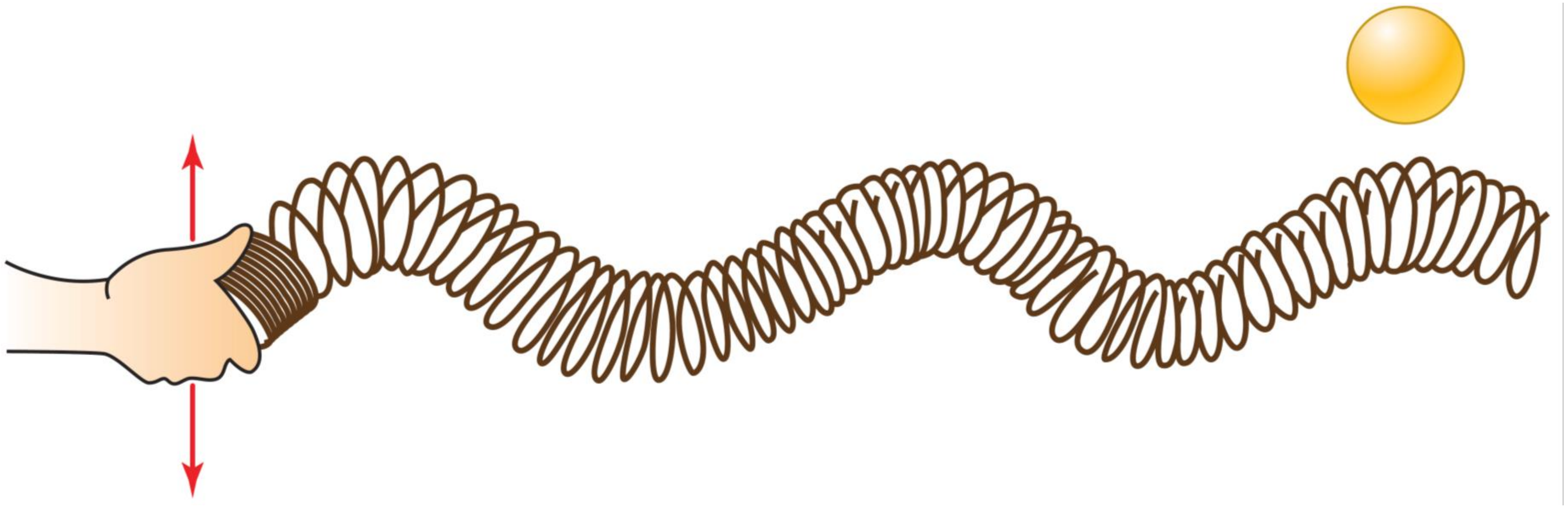
- **Amplitude (a) [m]:** the height of a crest or trough of a wave from neutral
- **Wavelength (λ) [m]:** the spacing between the crests (or troughs) of a wave
- **Frequency (f) [Hz = 1/s]:** the number of wavelengths that pass by a fixed point in one second
- **Period (T) [s]:** the time is takes one wavelength to pass by a fixed point

$$f = \frac{1}{T}$$



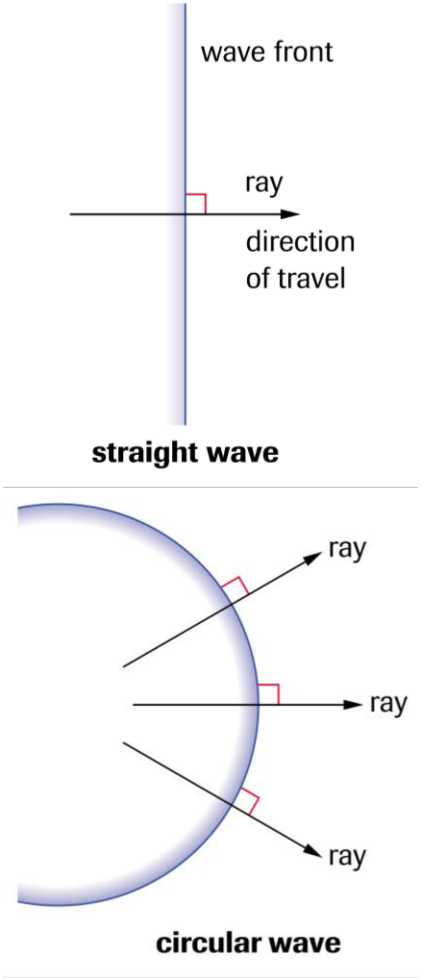
TRANSMISSION

- **Transverse Wave:** periodic disturbance where particles in the medium oscillate at right angles to the direction in which the wave travels



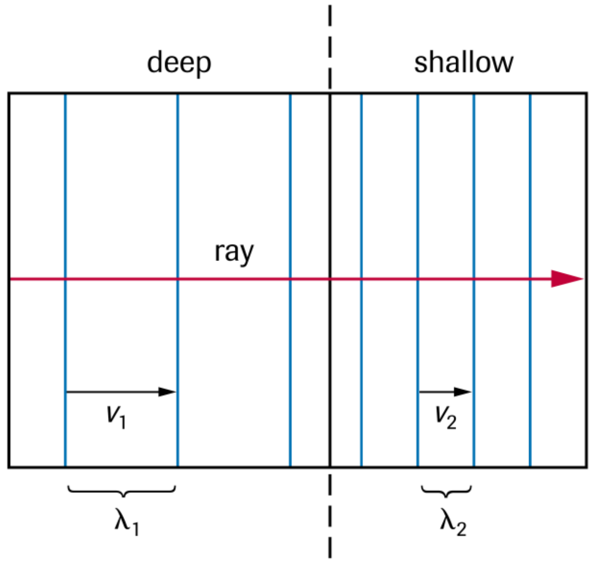
TRANSMISSION – CONT.

- Waves move away from the source
  - A linear source creates a straight wave
  - A point source creates a circular wave
- **Wave Front:** the leading edge of a continuous crest or trough
- **Wave Ray:** a straight line, drawn perpendicular to a wave front, indicating the direction of transmission



TRANSMISSION – CONT.

- For waves coming from a constant-frequency source, a change in medium will affect the wave properties
- Mediums that slow down a wave (for example: shallow water) will decrease its wavelength ($\lambda \propto v$)
- A wave with a higher frequency will also have a shorter wavelength ($f \propto \frac{1}{\lambda}$)



- **Universal Wave Equation:** the relationship between the speed, frequency and wavelength of a wave; applicable in all three dimensions

$$\boldsymbol{v = f\lambda}$$

TRANSMISSION – CONT.

- Consider a wave with constant frequency, f:
- For an initial conditions speed, v_1, we have

$$v_1 = f\lambda_1$$

- If we change the speed of the wave to v_2, we have

$$v_2 = f\lambda_2$$

- The ratio of the two conditions is

$$\frac{v_1}{v_2} = \frac{f\lambda_1}{f\lambda_2} = \frac{\lambda_1}{\lambda_2}$$

PROBLEM 1

A water wave has a wavelength of 2.0 cm in the deep section of a tank and 1.5 cm in the shallow section. If the speed of the wave in the shallow water is 12 cm/s, what is its speed in the deep water?

PROBLEM 1 – SOLUTIONS

$\lambda_1 = 2.0 \text{ cm}$

$\lambda_2 = 1.5 \text{ cm}$

$v_2 = 12 \text{ cm/s}$

$v_1 = ?$

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

$$v_1 = \left(\frac{\lambda_1}{\lambda_2}\right) v_2$$

$$= \left(\frac{2.0 \text{ cm}}{1.5 \text{ cm}}\right) 12 \text{ cm/s}$$

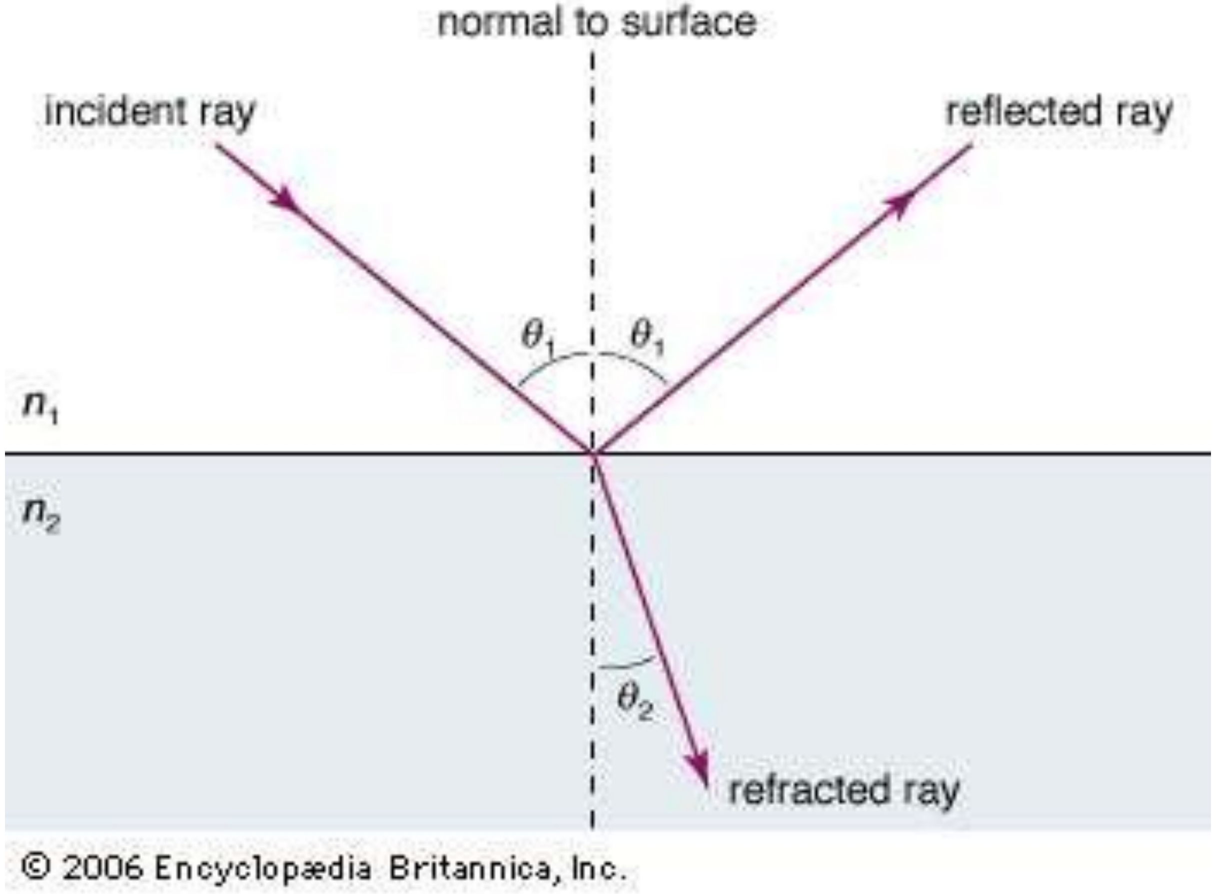
$$v_1 = 16 \text{ cm/s}$$

The speed of the wave in deep water is 16 cm/s.

RECALL – DEFINITIONS

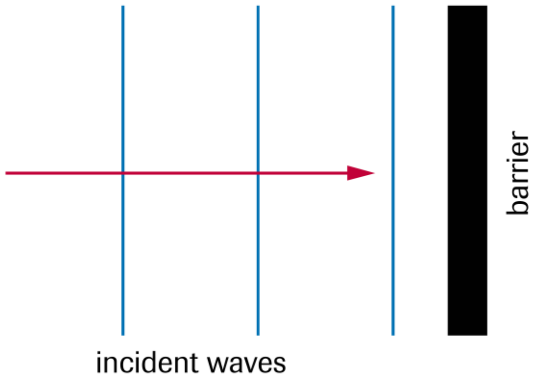
- **Normal:** a straight line drawn perpendicular to a barrier struck by a wave
- **Angle of Incidence (θ_i):** the angle between the incident wave front and the barrier, or the angle between the incident ray and the normal
- **Angle of Reflection (θ_r):** the angle between the reflected wave front and the barrier, or the angle between the reflected ray and the normal
- **Angle of Refraction (θ_R):** the angle between the normal and the refracted ray, or between the refracted wave front and the boundary

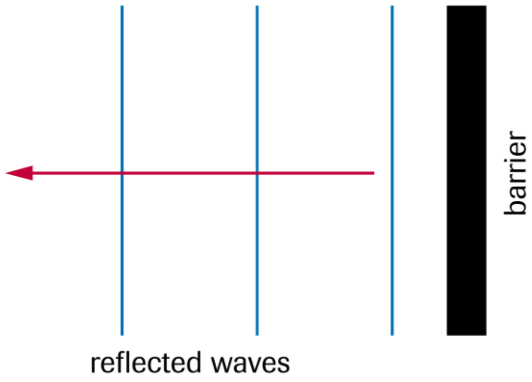
REFLECTION-REFRACTION OF WAVES

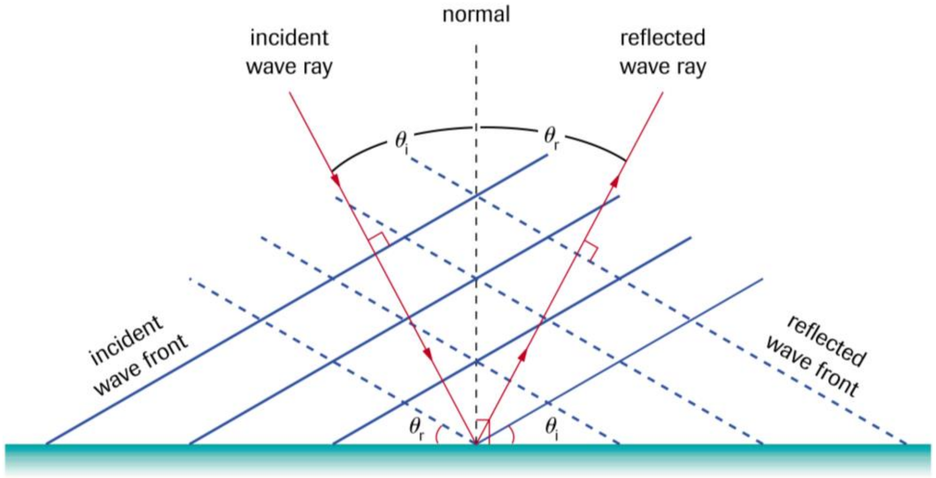


REFLECTION FROM A STRAIGHT BARRIER

- If a wave encounters a straight barrier head on (perpendicular) it is reflected back along its original path
- At any angle other than 90°, it is reflected at the same angle
- The speed, frequency and wavelength are unchanged

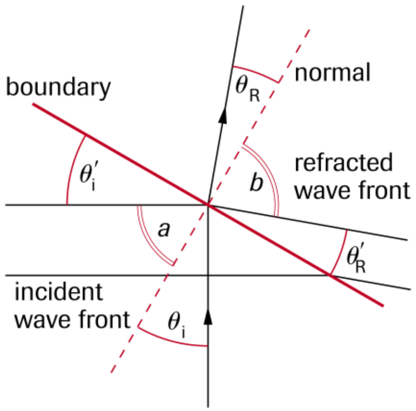


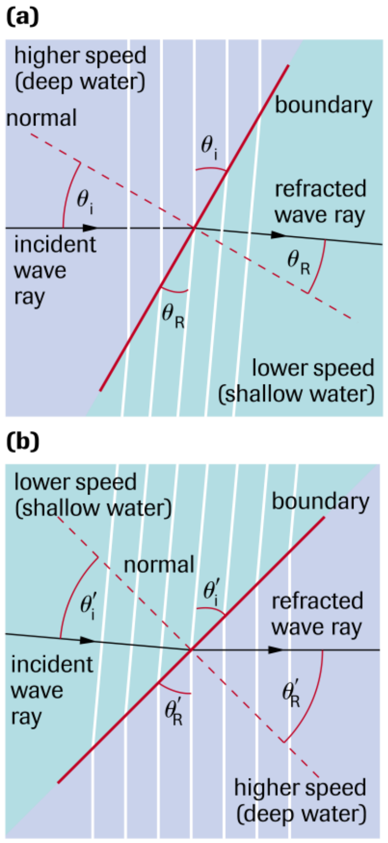




REFRACTION

- **Refraction:** the bending effect on a wave's direction that occurs when the wave enters a different medium at an angle
- When entering a medium where a wave slows down, the path bends towards the normal
- When entering a medium where the wave speeds up, the path bends away from the normal





REFRACTION – CONT.

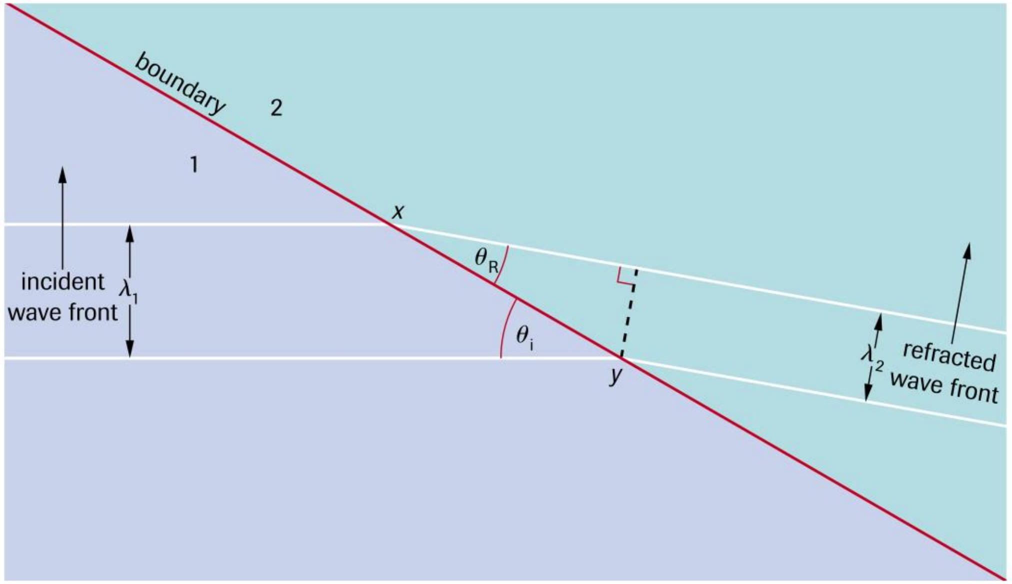
- Geometrically, we can see that

$$\sin \theta_i = \frac{\lambda_1}{xy}$$

and

$$\sin \theta_R = \frac{\lambda_2}{xy}$$

  - xy – the section of the boundary through which the wave passes



REFRACTION – CONT.

- The ratio of the sines gives us

$$\frac{\sin\theta_i}{\sin\theta_R} = \frac{\left(\frac{\lambda_1}{xy}\right)}{\left(\frac{\lambda_2}{xy}\right)} = \frac{\lambda_1}{\lambda_2}$$

- Since $\frac{\lambda_1}{\lambda_2}$ is constant, $\sin\theta_i \propto \sin\theta_R$, so

$$\sin\theta_i = n\sin\theta_R$$

$$\frac{\sin\theta_i}{\sin\theta_R} = n$$

  - n – index of refraction (proportionality constant)

REFRACTION – CONT.

- **Absolute Index of Refraction ($\boldsymbol{n}$):** the index of refraction for light passing from air or a vacuum into a substance

$$n = \frac{c}{v} = \frac{\sin\theta_i}{\sin\theta_R}$$

  - c – speed of light in a vacuum

Substance	Absolute Refractive Index
vacuum	1.000 000
air	1.000 29
ice	1.31
water	1.333
ethyl alcohol	1.36
turpentine	1.472
glass	1.50
Plexiglas	1.51
crown glass	1.52
polystyrene	1.59
carbon disulphide	1.628
flint glass	1.66
zircon	1.923
diamond	2.417
gallium phosphide	3.50

REFRACTION – CONT.

- Combining what we have learned about the refraction of waves and the Universal Wave Equation, we get

$$\frac{\boldsymbol{sin\,\theta_1}}{\boldsymbol{sin\,\theta_2}} = \frac{\boldsymbol{v_1}}{\boldsymbol{v_2}} = \frac{\boldsymbol{\lambda_1}}{\boldsymbol{\lambda_2}} = \frac{\boldsymbol{n_2}}{\boldsymbol{n_1}}$$

PROBLEM 2

A 5.0 Hz water wave, travelling at 31 cm/s in deep water, enters shallow water. The angle between the incident wave front in the deep water and the boundary between the deep and shallow regions is 50°. The speed of the wave in the shallow water is 27 cm/s. Find

(a) the angle of refraction in the shallow water

(b) the wavelength in shallow water

PROBLEM 2 – SOLUTIONS

(a) $f = 5.0\ \text{Hz}$ $\theta_1 = 50.0°$

$v_1 = 31\ \text{cm/s}$ $\theta_2 = ?$

$v_2 = 27\ \text{cm/s}$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

$$\sin\theta_2 = \left(\frac{v_2}{v_1}\right)\sin\theta_1$$

$$\sin\theta_2 = \left(\frac{27\ \text{cm/s}}{31\ \text{cm/s}}\right)\sin 50.0°$$

$$\theta_2 = 41.9,\ \text{or}\ 42°$$

The angle of refraction is 42°.

PROBLEM 2 – SOLUTIONS CONT.

(b) $$\lambda_2 = \frac{v_2}{f_2} \quad \text{but } f_2 = f_1 = 5.0 \text{ Hz}$$

$$= \frac{27 \text{ cm/s}}{5.0 \text{ Hz}}$$

$$\lambda_2 = 5.4 \text{ cm}$$

The wavelength in shallow water is 5.4 cm.

PROBLEM 3

For a light ray travelling from glass into water, find

(a) the angle of refraction in water, if the angle of incidence in glass is 30.0°

(b) the speed of light in water

$n_g = n_1 = 1.50$ $\theta_g = \theta_1 = 30.0°$

$n_w = n_2 = 1.333$ $\theta_w = \theta_2 = ?$

PROBLEM 3 – SOLUTIONS

(a)

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}$$

$$\frac{\sin \theta_g}{\sin \theta_w} = \frac{n_w}{n_g}$$

$$\frac{\sin 30.0°}{\sin \theta_w} = \frac{1.333}{1.50}$$

$$\sin \theta_w = \frac{1.50 \sin 30.0°}{1.333}$$

$$\theta_w = 34.3°$$

The angle of refraction in water is 34.3°.

PROBLEM 3 – SOLUTIONS CONT.

(b) $n_a = n_1 = 1.00$

$n_w = n_2 = 1.333$

$v_1 = c = 3.00 \times 10^8$ m/s

$v_2 = ?$

$$\frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$$v_2 = \frac{n_1 v_1}{n_2}$$

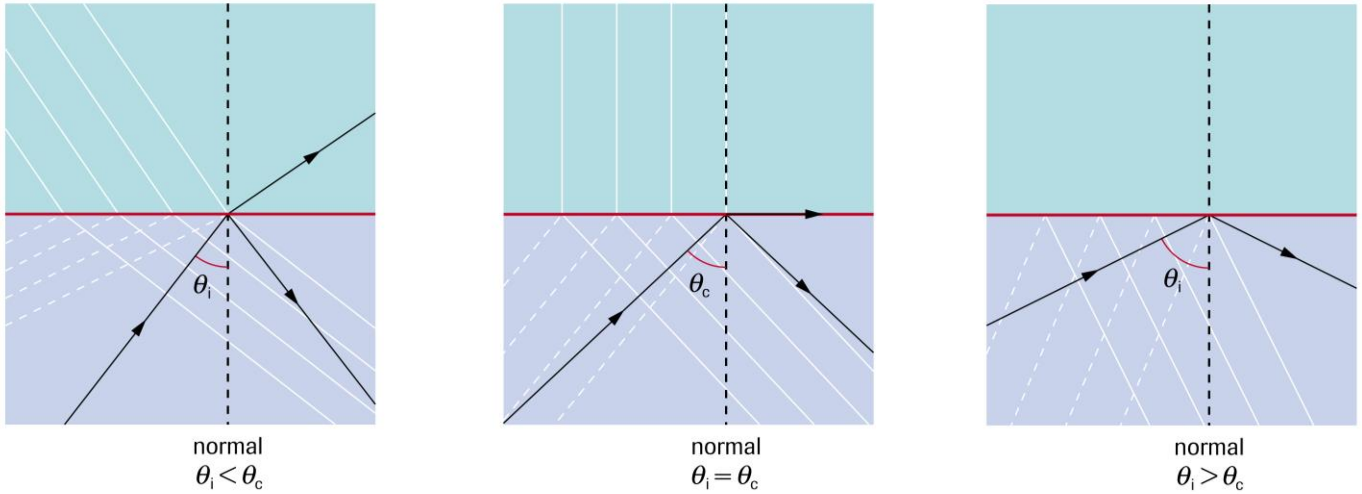
$$= \frac{(1.00)(3.00 \times 10^8 \text{ m/s})}{1.333}$$

$$v_2 = 2.26 \times 10^8 \text{ m/s}$$

The speed of light in water is 2.26×10^8 m/s.

PARTIAL REFLECTION-REFRACTION

- **Critical Angle (θ_c):** the angle of incidence on a less dense (faster) medium for which the angle of refraction is 90°
- **Total Internal Reflection:** the reflection of light in an optically denser medium; it occurs when the angle of incidence in the denser medium is greater than a certain critical angle



SUMMARY:
WAVES IN TWO DIMENSIONS

- The wavelength of a periodic wave is directly proportional to its speed.
- The frequency of a periodic wave is determined by the source and does not change as the wave moves through different media or encounters reflective barriers.
- All periodic waves obey the universal wave equation, $v = f\lambda$.
- The index of refraction for a pair of media is the ratio of the speeds or the ratio of the wavelengths in the two media $\left(\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}\right)$.
- Snell's law $\left(n = \frac{\sin\theta_i}{\sin\theta_R}\right)$ holds for waves and for light.
- When a wave passes from one medium to another, the wavelength changes and partial reflection–partial refraction can occur.

PRACTICE

Readings

- Section 9.1 (pg 444)

Questions

- pg 452 #1,3,5,7,8